Solving and Analysing Simple Structural Problems

Any simple structural problem can be analysed or solved using a few steps. These are:

1) Create a complete Free Body Diagram to visualize the problem
2) Choose the equations that will help solve the problem
3) Isolate the unknowns (this section also includes an example about subtracting one equation from another)
4) Check your answer if you are unsure

<u>On any exam following through on these steps will at least get you part marks even if your answer is incorrect.</u>

Each of these steps is described in more detail below.

Step 1: Create a complete Free Body Diagram

A good Free Body Diagram is the key to understanding (and solving) a structural problem. A complete Free Body Diagram will include:

1) All forces (and their locations) acting on the structure – and converted (if necessary) into concentrated loads
2) All of the different kinds of connectors supporting the structure
3) All of the reactions (and their locations) at those connectors

The best way to understand these aspects of a Free Body Diagram is with an example. Here's a typical problem you could be presented with:

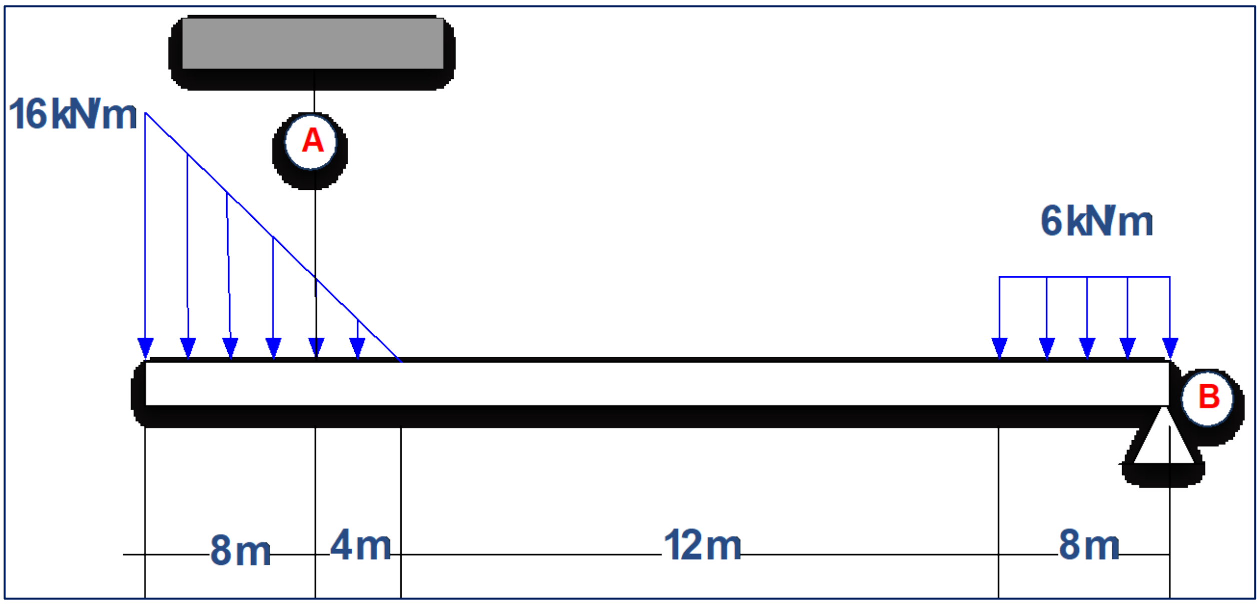

Figure 1: Problem 1

As you can see, this structure is supported by a hanger at A and a pin joint at B. It also has a uniform load of 6 kN/m and a uniformly varying load of 16 kN/M. From your work with connectors you know that a pin joint has two reactions (B_x and B_y) and a hanger has just one (A_y). Mark these on your free body diagram. These are the reactions you will be solving for.

Next convert the uniform load and the uniformly varying load into concentrated loads.

To convert a uniform load first calculate the total load which this case is the load per metre times the number of metres that it acts over or 6 kN/m x 8 m or 48 kN. This concentrated force will be positioned half way along the length of the uniform load or 8/2 m or 4 m from point B.

A uniformly varying load is a little more complicated. The total load is derived by multiplying the load (16 kN/m) times the distance over which its acts (12 m) and then dividing by 2 or (16 kN/m x 12 m)/2 = 192/2 = 96 kN.

The equivalent concentrated load will act a point that is one third the distance over which the uniformly distributed load acts BUT measured from the high end (or greatest magnitude) of that force. In this case 12 m/3 gives 4 m from the left end of the beam.

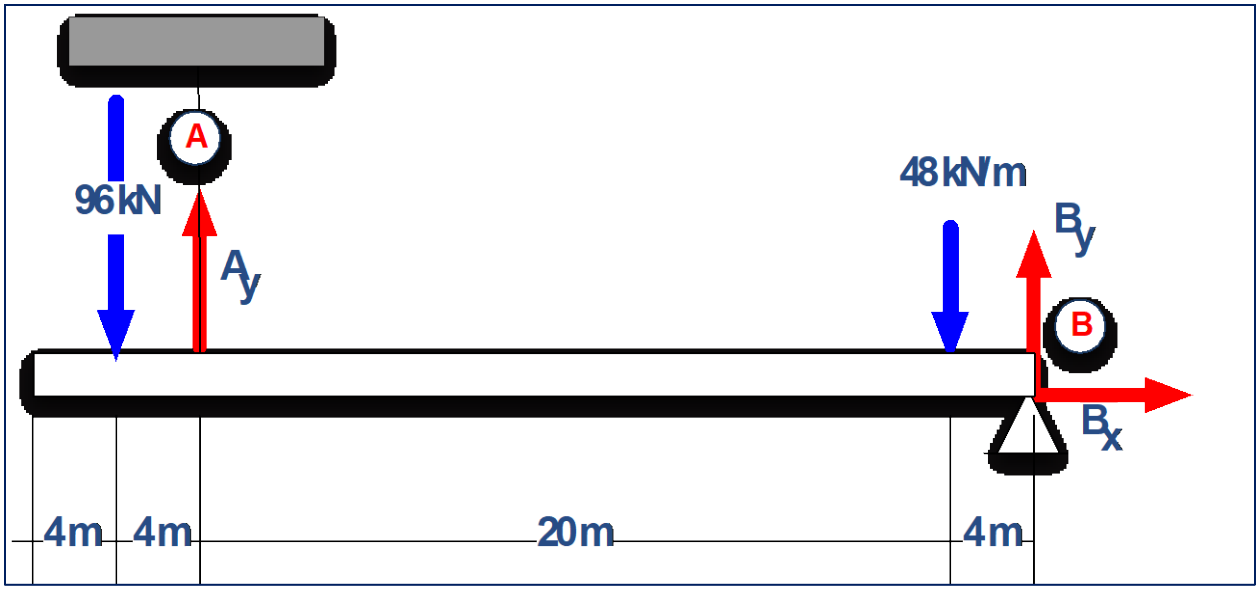

Figure 2: Free Body Diagram for Problem 1

Mark these two concentrated loads on your Free Body Diagram. With this diagram you should be able to analyse and solve for all the unknown reaction.

For more about Common Loads on Beams see http://www.learnaboutstructures.com/Common-Load-Types-for-Beams-and-Frames

Step 2: Choose the Equations that define the problem

In most of these structural analysis questions you will use the 3 equilibrium equations that you learned earlier.

The equilibrium equations are:
∑ of moment forces must equal zero
∑ of forces along the y axis must equal zero
∑ of forces along the x axis must equal zero

In other instances, however, you may also need to use trigonometric functions (to break an angled vector into its x and y components) or the Pythagorean Theorem or Moment Couples (for eccentric loads). See the Open Educational Resources for each of these concepts.

Looking at the free body diagram we can see that in this case we need to find the values of B_y, B_x and A_y.

The first thing we can observe is that there are no forces or components of forces acting in the horizontal direction so we know that for the ∑ of forces along the x axis to equal zero then B_x must also equal zero. This is not always the case but here it does simplify things since we only need to solve for B_y and A_y.

Because both B_y and A_y are vertical reactions and both are unknown this means that the equations for the ∑ of forces along the y axis will have two unknowns (B_y and A_y) which is difficult to solve so we'll start with the moment equations.

TIP: When choosing your equations pick locations that will simplify your calculations. For example if you sum the moment about B in the example above the reactions (B_x and B_y) drop out of the equations because the distance between them and the point B is 0.

Step 3: Isolate the Unknowns: Part 1

To solve any equation, you need to get the unknown (such as B_y) by itself on one side of the equal sign. This is what isolate the unknowns means.

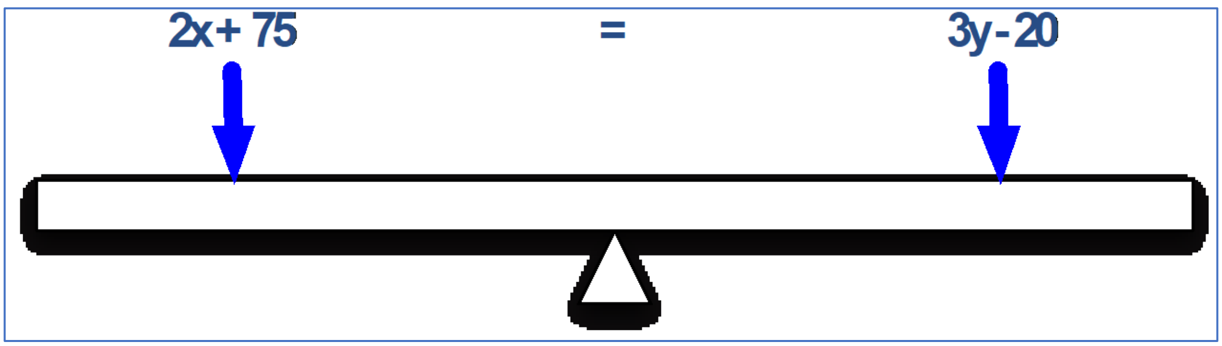

Figure 3: Think of an equation like a beam sitting on a pin joint with two balanced forces acting it. The equal sign is like the pin joint. In order to keep the beam from moving, whatever you do to one side of the beam you must do to the other. If you add 20 kg to the left side, then you must add 20 kg to the right side to keep it balanced. Similarly, to keep the equation balanced if you add 20 to the left side of the equation then you must add 20 to the right side.

You should imagine that an equation (any equation) is like a balance scale (as shown above). Things are equal when what's on one side is equal to or balanced by what's on the other. In this sense, the '=' is like the fulcrum of a balance scale or a seesaw.

This is the critical thing to remember – to keep the equation 'balanced' what you do to one side of the equal sign you have to do to the other side as well. This is the secret to solving equations because there all sorts of things you can do to the pieces of an equation. You can subtract 18 from both sides or add 34 to both sides. You can multiply both sides by 2 or divide by .0786. You can square both sides or take the square root of both sides. There is an endless set of possibilities but the reason you are doing these things is to isolate the unknowns on one side or the other.

In general, you can approach this in 4 steps:

1) Substitute in all the values you know
2) Simplify the equation by performing any obvious operations
3) Add or subtract values from both sides of the equation
4) Divide or multiply both sides of the equation

The end result should be that the unknown is isolated.

Now we can apply these steps to the problem we have been working on:

We can start with the Moments about A and we know that $\sum M_A = 0$ so:

Step 1: Substitute in all the values you know) $B_y(24\text{ m}) - 48\text{ kN/m}(20\text{m}) + 96\text{ kN/m}(4\text{m}) = 0$

Step 2: Simplify by performing basic operations) $B_y(24) - 960\text{ kN} + 384\text{ kN} = 0$

$B_y(24) - 576\text{ kN} = 0$

Step 3: Add or Subtract, + 576 to both sides) $B_y(24) = 576\text{ kN}$

Step 4: Divide of Multiply, ÷ both sides by 24) $B_y = 24\text{ kN}$

Now that we know B_y we can move right away to the $\sum$ of forces along the y axis which is:

$A_y + B_y - 96\text{ kN} - 48\text{ kN} = 0$

Step 1: $A_y + 24\text{ kN} - 96\text{ kN} - 48\text{ kN} = 0$

Step 2: $A_y - 120\text{ kN} = 0$

Step 3: $A_y = 120\text{ kN}$

(There was no need for Step 4 in this one)

Step 4: Check Your Answer

You don't have to do this but you can then check the correctness of your answer by seeing if the $\sum M_B = 0$

This gives

Step 1: -4m(48 kN)+ 24m(A_y) – 28m(96 kN) = 0

Step 2: -192 kNm + 24m(A_y) – 2688 kNm = 0

Step 2: 24m(A_y) – 2880 kNm = 0

Step 3: 24m(A_y) = 2880 kNm

Step 4: A_y = 120 kN which is correct

Step 3: Isolate the Unknowns: Part 2, Subtracting one Equation from Another

As we have noted in a number of locations, to solve a structural system you need to have as many equations as you do unknowns. Typically we deal with two equations and two unknowns or three equations and three unknows.

For example, you may have a situation with two equations such as:

$x - 16y = 32$ and $3x + 5y = 80$

Here the two unknowns are x and y.

One way to solve such a system is to subtract one equation from another. This is just another way of isolating the unknowns.

The following diagram shows a structural situation that can be solved by subtracting one equation from another. Here we see two bars with pin joints and a hinge supporting a 20 kN load.

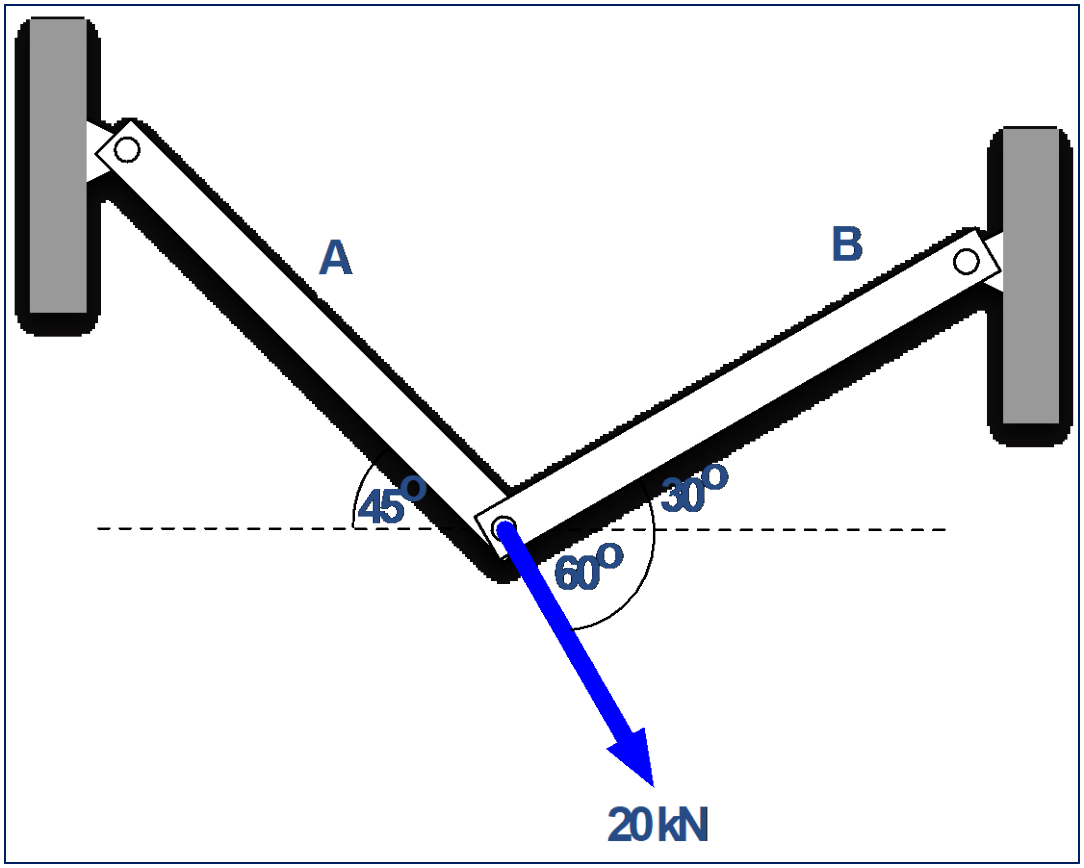

Figure 4: Problem 2

We can then map the system into an x-y coordinate system and draw a free body as shown below.

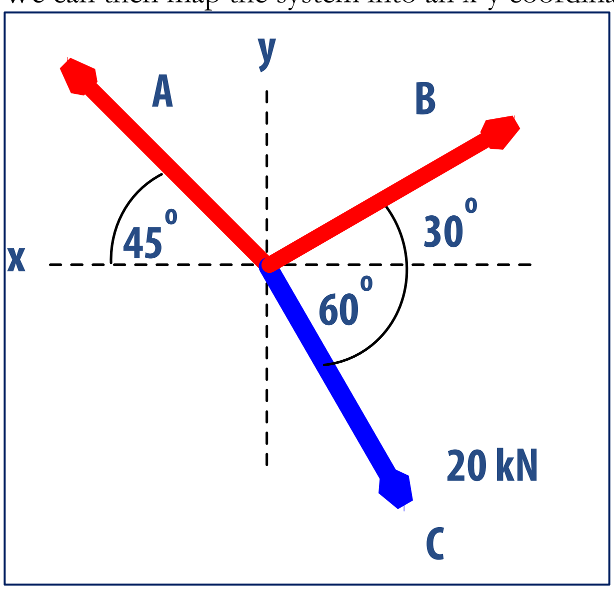

Figure 5: Free Body Diagram Problem 2

We can then break down each vector into its horizontal (x) and vertical (y) components as shown below.

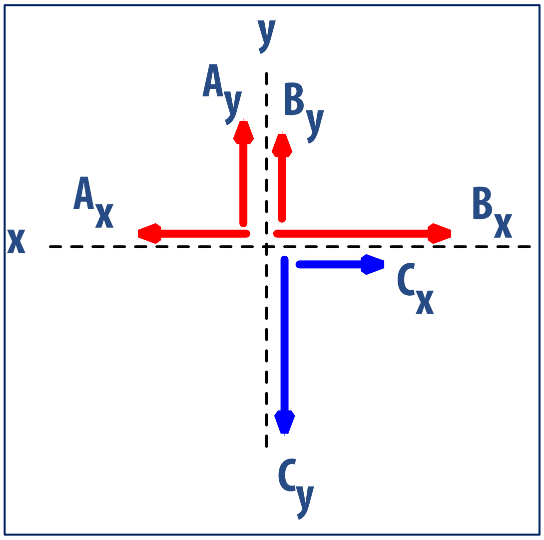

Figure 6: X and y components of Problem 2

Since none of the joints can prevent a rotational movement we don't have any moments and we can focus on the other two equilibrium equations:

$\sum$ of forces along the y axis must equal zero
$\sum$ of forces along the x axis must equal zero

In other word:
$\sum F_x = 0 = -A_x + B_x + C_x$ (Positive is to the right; Negative to the left)

To figure out the 'x' components of each of these vectors we'll need some of the trigonometric functions. Since Cos θ = Adjacent/Hypotenuse or Cos θ = x/Hypotenuse then

x = Cos θ x Hypotenuse

We can then substitute this information into $\sum F_x = 0 = A_x – B_x – C_x$

$\sum F_x = 0 = -\cos(45)A + \cos(30)B + \cos(60)C$

Substituting in the values we know gives:

-.71A + .87B + .5(20) =0

Or

-.71A + .87B +10 =0 (We'll call this Equation 1)

Similarly,

$\sum F_y = 0 = A_y + B_y – C_y$ (Positive is up: Negative is down)

To figure out the 'y' components of each of these vectors we'll again need some of the trigonometric functions. Since Sin θ = Opposite/Hypotenuse or Sin θ = y/Hypotenuse then

y = Sin θ x Hypotenuse

We can then substitute this information into $\sum F_y = 0 = A_y - B_y - C_y$

$\sum F_y = 0 = \sin(45)A + \sin(30)B - \sin(60)C$

Substituting in the values we know gives:

$.71A + .5B - .87(20) = 0$

Or

$.71A + .5B - 17.4 = 0$ (We'll call this Equation 2)

Collecting our two equations gives:
$-.71A + .87B + 10 = 0$
$.71A + .5B - 17.4 = 0$

You can see that if we add the two equations together the A pieces will cancel each other out and we will be left with just B.

$-.71A + .87B + 10 = 0$
$\underline{.71A + .5B - 17.4 = 0}$
$1.37B - 7.4 = 0$

Or

$1.37B = 7.4$

Or

$B = 7.4/1.37 = 5.4$ kN

We can then substitute in this value for B into one of our equations. Either one would work but we'll use equation 2:

$.71A + .5(5.4) - 17.4 = 0$

Or

$.71A + 2.7 - 17.4 = 0$

$.71A - 14.7 = 0$

A = 14.7/.71 = 20.7 kN

However, this way was a little too easy because the A's cancelled out. Instead we can also solve for A instead of B. To do this we'll need to multiply one of the equations by a factor that will make the B's cancel out. Again we start with the two equations:

-.71A + .87B +10 =0
.71A + .5B – 17.4 = 0

If we multiply equation 1 by .5/.87 or .57 then this will get us moving in the right direction. Remember, however, we need to make sure that every term in equation 1 is multiplied by .57. This gives:

-.71(.57)A + .87(.57)B + 10(.57) = 0(.57)

Or

-.40A + .5B + 5.7 = 0

We can now subtract this equation from equation 2.

.71A + .5B – 17.4 = 0
-.40A + .5B + 5.7 = 0
1.1A +0B – 23.1 = 0

Or
1.1A = 23.1

Or

A = 23.1/1.1 = 21 kN

(It's not identical to the value we calculated earlier but that's because of the approximations we used for the trigonometric functions but it is very close)

We can now plug this value into one of our equations to solve for B. Since we used equation 2 last time we'll use equation 1 this time.

-.71A + .87B +10 =0

Or

-.71(21) + .87B +10 = 0

-14.91 + .87B + 10 = 0

.87B -4.91 = 0

$.87B = 4.91$

$B = 4.91/.87 = 5.6$ kN

Again this isn't identical to the other calculations but it's close.
This is important: <u>Many of our calculations are approximations and always will be.</u>

Why does this work?
Subtracting (or adding) one equation from another works because of the balance beam analogy. In effect we are subtracting (or adding) the same amount to both sides of the equation.

Again, here are the two equations:

$-.71A + .87B + 10 = 0$
$.71A + .5B – 17.4 = 0$

In effect what we are doing is adding $.71A + .5B – 17.4$ to both sides of equation 1. Because $.71A + .5B – 17.4$ is equal to 0 (and vice versa) on the left side of the equation we are adding $.71A + .5B – 17.4$ and on the right side we are adding 0.

You can also think of this in terms of graphs in the x-y plane. It's easy to rephrase the equations as:

$-.71x + .87y + 10 = 0$
$.71x + .5y – 17.4 = 0$

Each of these equations defines a straight line that intersect each other at a single point (which is the solution to the problem). In this approach, the x value (or the A value) is a horizontal line that runs through the point of intersection and the y value (or the B value) is a vertical line that runs through the same point.